Backstepping Control of Induction Motors

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Abstract-
In this paper, a novel field-oriented induction motor drive using backstepping control design is presented. Backstepping control is proposed for replacing the existing PI controller to obtain high performance motion control systems, for the speed, flux and currents control loops. Stability analysis based on Lyapunov theory is also performed to guarantee the convergence of the speed tracking error from all possible initials conditions. Also, the computer simulations confirm that the proposed backstepping control scheme offers improved performance in terms of the trajectory tracking ability to time-varying reference input and robustness against parameters variation.

Keywords: Induction motor, field-oriented control, Backstepping control, Lyapunov theory.

I. INTRODUCTION

Induction motor drives, controlled by field oriented technique, have been widely used in industrial applications because their low cost, high reliability, power efficiency and easy maintenance. Induction motors are difficult to control for several reasons: (1) their dynamics are intrinsically non-linear and multivariable, (2) not all of the state variables and not all of the outputs to be controlled may be available for feedback; (3) there are critical parameters (for instance, load torque, stator and rotor resistances) which may considerably vary during operations. The concept of field orientation can be viewed as a nonlinear feedback transformation that achieves torque-flux decoupling technique [1]. More recently, various variations and improvements have been made for this control [2-4], which is based on a PI controller. In this approach, PI controllers are used in both the speed and inner flux and currents control loops. In many motion control applications, the PI controller works well and presents certain acceptable performance. However, when the system parameter uncertainties and
mismatch becomes significant due to load disturbances, it is difficult to achieve satisfactory performance based on the classical PI scheme.

On the other hand, the PI controller strategy does not consider the cross-relation between the outer and inner control loops, which essentially limits its performance.

The backstepping algorithm, which is used to replace the PI controller, presents very good position tracking response as well as rejection to load disturbance. In the past decade, research about backstepping control has been increased [5-16]. The backstepping theory is a systematic and recursive design methodology for nonlinear feedback control. In many cases, the feedback linearization method using geometric approach is only valid in some local region and with a disturbance-free setting. The backstepping design alleviates some of these limitations [5]. Moreover, the backstepping design offers a choice of design tools for accommodation of uncertainties and nonlinearities and can avoid wasteful cancellations. In addition, the backstepping control approach is capable of keeping almost all the robustness properties [6], [8-10].

In this paper, we propose an approach that combines field orientation principle and backstepping design. The idea of backstepping design is to select recursively some appropriate functions of state variables, in our case the speed and flux, as pseudo-control inputs for lower dimension subsystems of the overall system. When the procedure terminates, a feedback design for the true control input results which achieves the original design objective by virtue of a final Lyapunov function, which is formed by summing up the Lyapunov functions associated with each individual design stage. The paper is organized as follows. Section 2 gives the induction motor model in field-oriented control. Then, the backstepping control design with Lyapunov theory is shown in section 3. The simulation results presented in section 4 for an induction motor verify the validity of the proposed control. Finally, some conclusions are given in section 5.

II. INDUCTION MOTOR MODEL

The dynamic model of IM in ($\alpha, \beta$) stationary reference frame, which includes both the electrical and mechanical dynamics, is a fifth order system of nonlinear equations and can be described by the following differential equations.

$$\frac{d\Omega}{dt} = \frac{P.M}{JL_s} (\phi_{r,\alpha}i_{\beta,\alpha} - \phi_{r,\beta}i_{\alpha,\alpha}) - \frac{C_r}{J}$$
\[ \frac{d\phi_{\alpha}}{dt} = -\frac{R}{L_r} \phi_{\alpha} - P\Omega \phi_{\beta} + \frac{R}{L_r} M j_{\alpha} \]

\[ \frac{d\phi_{\beta}}{dt} = -\frac{R}{L_r} \phi_{\beta} + P\Omega \phi_{\alpha} + \frac{R}{L_r} M j_{\beta} \]

\[ \frac{d}{dt} \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} = \frac{M.R_c}{\sigma L_s_L_r} \phi_{\alpha} + \frac{P.M}{\sigma L_s_L_r} \Omega \phi_{\beta} + \frac{M^2.R_c + L_s^2.R_r}{\sigma L_s.L_r^2} j_{\alpha} + \frac{1}{\sigma L_r} V_{\alpha} \]

\[ \frac{d}{dt} \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} = \frac{M.R_c}{\sigma L_s.L_r} \phi_{\beta} - \frac{P.M}{\sigma L_s.L_r} \Omega \phi_{\alpha} - \frac{M^2.R_c + L_s^2.R_r}{\sigma L_s.L_r^2} j_{\beta} + \frac{1}{\sigma L_r} V_{\beta} \]

\[ \begin{array}{c}
\frac{d\phi_{\alpha}}{dt} + P\Omega \phi_{\beta} + \frac{R}{L_r} M j_{\alpha} \\
\frac{d\phi_{\beta}}{dt} - P\Omega \phi_{\alpha} + \frac{R}{L_r} M j_{\beta}
\end{array} \]

The above model for the induction motor is obviously a highly coupled multivariable nonlinear system. It is very difficult to control such a system directly based on this model. According to the vector control principle, the \( q \)-axis flux \( \phi_{rq} \) is always forced to be zero in order to orient all the rotor flux in the \( d \)-axis and a decoupled system for linear torque control is achieved. This control technique involves a transformation of the representation for the statevector in the fixed stator frame \( (\alpha,\beta) \) into that in a frame \( (d, q) \).

The stator field angle \( \theta_s \) is estimated as:

\[ \theta_s = \arctan \left( \frac{\phi_{r\beta}}{\phi_{r\alpha}} \right) \]

Transformation between the two different frames is:

\[ \begin{bmatrix} x_{sa} \\ x_{sb} \end{bmatrix} = \begin{bmatrix} \cos(\theta_s) & \sin(\theta_s) \\ -\sin(\theta_s) & \cos(\theta_s) \end{bmatrix} \begin{bmatrix} x_{sd} \\ x_{sq} \end{bmatrix} \]

(3)

Where \( x \) can be used for current, flux \( \phi \) and voltage \( v \). Using equation (2), we obtain:

\[ \phi_r = \phi_{r\alpha} + j \phi_{r\beta} = \sqrt{\phi_{r\alpha}^2 + \phi_{r\beta}^2} \left[ \frac{\phi_{r\alpha}}{\sqrt{\phi_{r\alpha}^2 + \phi_{r\beta}^2}} + j \frac{-\phi_{r\beta}}{\sqrt{\phi_{r\alpha}^2 + \phi_{r\beta}^2}} \right] \]

\[ \phi_r = \sqrt{\phi_{r\alpha}^2 + \phi_{r\beta}^2} \left[ \cos(\theta_s) + j \sin(\theta_s) \right] \]

Therefore the equations (3) become:

\[ i_{sd} = \frac{\phi_{r\alpha} j_{sa} + \phi_{r\beta} j_{sb}}{\sqrt{\phi_{r\alpha}^2 + \phi_{r\beta}^2}} \]

(5)
\[ i_{sq} = \frac{\phi_{ra}i_{s\beta} - \phi_{r\beta}i_{s\alpha}}{\sqrt{\phi_{ra}^2 + \phi_{r\beta}^2}} \]  
\[ (6) \]

\[ \phi_{rd} = \sqrt{\phi_{ra}^2 + \phi_{r\beta}^2} \]

\[ \phi_{rq} = 0 \]

\[ V_{sd} = \frac{V_{sa}\phi_{ra} + V_{s\beta}\phi_{r\beta}}{\sqrt{\phi_{ra}^2 + \phi_{r\beta}^2}} \]  
\[ (8) \]

\[ V_{sq} = \frac{V_{s\beta}\phi_{ra} - V_{sa}\phi_{r\beta}}{\sqrt{\phi_{ra}^2 + \phi_{r\beta}^2}} \]  
\[ (9) \]

Using this transformation, the state equations (1) can be rewritten in the new state variables as:

\[ \frac{d\Omega}{dt} = \frac{\mu}{J} \phi_{rd}i_{sq} - \frac{C_r}{J} \]

\[ \frac{d\phi_{rd}}{dt} = -\alpha\phi_{rd} + \alpha M i_{sd} \]

\[ \frac{di_{sd}}{dt} = -\eta i_{sd} + \alpha \beta \phi_{rd} + P\Omega i_{sq} + \alpha M \frac{i_{sq}^2}{\phi_{rd}} + \frac{1}{\sigma L_s} V_{sd} \]  
\[ (10) \]

\[ \frac{di_{sq}}{dt} = -\eta i_{sq} - \beta P\Omega \phi_{rd} + P\Omega i_{sd} - \alpha M \frac{i_{sq} i_{sd}}{\phi_{rd}} + \frac{1}{\sigma L_s} V_{sq} \]

\[ \frac{d\theta_s}{dt} = P\Omega + \alpha M \frac{i_{sq}}{\phi_{rd}} \]

Where:

\[ \mu = \frac{PM}{L_r}, \quad \alpha = \frac{R_r}{L_r}, \quad \eta = \frac{M^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} \quad \text{and} \quad \beta = \frac{M}{\sigma L_s L_r} \]

Using the decoupled control approaches, the dynamic behavior of the induction motor is rather similar to that of a separately excited DC motor. However, the decoupled relationship is obtained by means of a proper selection as state co-ordinates, under the hypothesis that the rotor flux is kept constant [1], [5] and [10]. Therefore, the rotor speed is only...
asymptotically decoupled from rotor flux, and the speed is linearly related to torque current only after the rotor flux becomes the steady-state values. So, the induction motor system (10) leads a simplified system structure with two approximately decoupled subsystems. The first one is a subsystem with state vector \((\Omega, i_{sq})\) and control \(V_{sq}\), and the second one with \((\phi_d, i_{sd})\) as state and \(V_{sd}\) as control input. Particularly, this structure allows as to conveniently applying backstepping design techniques to replace the traditional nonlinear feedback PI control of the field oriented control technique for better performance. Thus, we will take a different path than the linearizing control of the field oriented control technique. The subsystem structure will be fully exploited in our control design as detailed in the next section.

### III. BACKSTEPping CONTROL

The backstepping is a systematic and recursive design methodology for nonlinear feedback control. The backstepping design offers a choice of design tools for accommodation of uncertainties and nonlinearities and can avoid wasteful cancellations. The idea of backstepping design is to select recursively some appropriate functions of state variables as pseudo-control inputs for lower dimension subsystems of the overall system. Each backstepping stage results in a new pseudo-control design from preceding design stages. When the procedure terminates, a feedback design for the true control input results which achieves the original design objective by virtue of a final Lyapunov function, which is formed by summing the Lyapunov functions associated with each individual design stage[14-16]. The backstepping design procedure consists of the following three steps.

**Step 1**

This first step consists in identifying the errors \(z_1\) and \(z_2\) which respectively represent the error between real speed \(\Omega\) and reference speed \(\Omega_{ref}\), as well as between the rotor flux module \(\phi_d\) and its reference \(\phi_{ref}\)

\[
\begin{align*}
z_1 &= \Omega_{ref} - \Omega \\
z_2 &= \phi_{ref} - \phi_d \\
\end{align*}
\]

The derivative of (11) is computed as

\[
\begin{align*}
\dot{z}_1 &= \dot{\Omega}_{ref} - \dot{\Omega} = \dot{\Omega}_{ref} - \frac{\mu}{j} \phi_d i_{sq} + \frac{T_L}{j} \\
\dot{z}_2 &= \dot{\phi}_{ref} - \dot{\phi}_d = \dot{\phi}_{ref} + \tau_r \phi_d - \tau_r M i_{sd} \\
\end{align*}
\]

The first Lyapunov candidates \(v_1\) is chosen as
\[ v_1 = \frac{1}{2} (z_1^2 + z_2^2) \]

So, the derivative of (13) is computed as
\[
\dot{v}_1 = z_1 \left( \dot{\Omega}_{ref} - \frac{\mu}{j} \Phi_d i_{sq} + \frac{T_L}{j} \right) + z_2 \left( \dot{\Phi}_{ref} + \tau_r \Phi_d - \tau_r M i_{sd} \right)
\]

Thus, the tracking objectives will be satisfied if we choose
\[
(i_{sq})_{ref} = \frac{1}{\Phi_d \mu} \left( k_1 z_1 + \dot{\Omega}_{ref} \right) + \frac{T_L}{\mu}
\]
\[
(i_{sd})_{ref} = \frac{1}{\tau_r M} \left( k_2 z_2 + \dot{\Phi}_{ref} + \tau_r \Phi_d \right)
\]

Where \( k_1 \) and \( k_2 \) are positive design constants that determine the closed loop dynamics. Then (12) can be expressed as
\[
\dot{z}_1 = -k_1 z_1
\]
\[
\dot{z}_2 = -k_2 z_2
\]

Therefore, (14) can be rewritten as
\[
\dot{v}_1 = -k_1 z_1^2 - k_2 z_2^2 < 0
\]

So, the control \((i_{sq})_{ref}\) and \((i_{sd})_{ref}\) in (15) is asymptotically stabilizing.

**Step 2**

Define other errors signals between the current and reference currents.
\[
z_3 = (i_{sq})_{ref} - i_{sq}
\]
\[
z_4 = (i_{sd})_{ref} - i_{sd}
\]

With this definition, (12) can be expressed as
\[
\dot{z}_1 = -k_1 z_1 + \frac{\mu}{j} z_3
\]
\[
\dot{z}_2 = -k_2 z_2 + \tau_r M z_4
\]

From (18), the errors dynamics are given by
\[
\dot{z}_3 = (i_{sq})_{ref} - i_{sq}
\]
\[
\dot{z}_4 = \frac{1}{\alpha M} \left( k_z \dot{z}_2 + \dot{i}_{\text{ref}} + \alpha \phi_d \right) - i_{\text{sd}}
\]

\[=
(i_{\text{dc}})_{\text{ref}} - \delta_1 - \frac{1}{\sigma L_s} V_{sq}
\]

Where:

\[
\delta_1 = -\eta i_{sq} - \beta p\Omega\dot{\phi}_d - p\Omega i_{sd} - \alpha M \frac{i_{sq} i_{sd}}{\phi_d}
\]

\[
\delta_2 = -\eta i_{sd} + \alpha \beta \phi_d + p\Omega i_{sq} + \alpha M \frac{i_{sq}^2}{\phi_d}
\]

**Step 3**

Since the actual control inputs \(V_{sd}, V_{sq}\) have appeared in the above equations, we can go to the final step. Now, we define the following Lyapunov function candidate.

\[
v_2 = \frac{1}{2} \left( z_1^2 + z_2^2 + z_3^2 + z_4^2 \right)
\]

Taking the time derivative of \(v_2\), we obtain

\[
\dot{v}_2 = z_1 \ddot{z}_1 + z_2 \ddot{z}_2 + z_3 \ddot{z}_3 + z_4 \ddot{z}_4
\]

This equation can be rewritten in the following from

\[
\dot{v}_2 = -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 - k_4 z_4^2 + z_3 \left( k_3 z_3 + (i_{sq})_{\text{ref}} - \delta_1 - \frac{1}{\sigma L_s} V_{sq} \right)
\]

\[
+ z_4 \left( k_4 z_4 + (i_{sq})_{\text{ref}} - \delta_2 - \frac{1}{\sigma L_s} V_{sd} \right)
\]

The choice of \(k_3 > 0\) and \(k_4 > 0\) can be made such that \(\dot{v}_2 < 0\). At last, in order to make the derivative of the complete Lyapunov function (23) be negative definite, the d-axis and q-axis voltage control input is chosen as follows.

\[
V_{sd} = \sigma L_s (i_{sd})_{\text{ref}} + k_4 z_4 - \delta_2
\]

\[
V_{sq} = \sigma L_s (k_3 z_3 + (i_{sq})_{\text{ref}} - \delta_1)
\]

Then, (20) can be expressed as
\[ \begin{align*} 
\dot{x}_3 &= -k_3 x_3 - x_1 \frac{\mu}{J} \\
\dot{x}_4 &= -\alpha M z_2 - k_4 z_4 
\end{align*} \]

To show boundedness of all states, we can rearrange the dynamical equations from (19) and (25) as

\[
\begin{bmatrix} 
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3 \\
\dot{z}_4
\end{bmatrix} = 
\begin{bmatrix} 
-k_1 & 0 & \frac{\mu}{J} & 0 \\
0 & -k_2 & 0 & \alpha M \\
-\frac{\mu}{J} & 0 & -k_3 & 0 \\
0 & -\alpha M & 0 & -k_4
\end{bmatrix} 
\begin{bmatrix} 
z_1 \\
z_2 \\
z_3 \\
z_4
\end{bmatrix} = AZ
\]

Where \( A \) can be shown to be Hurwitz, this proves the boundedness of all the states.

The block diagram of the proposed backstepping control scheme is presented in figure (1). The blocks ‘\( i_{\text{sdref}} \)’ calculation and ‘\( i_{\text{sqref}} \)’ calculation provide the currents references from the rotor flux and speed errors, through the equation (15) which represent the fictive control. The voltage command based on currents errors are given by the two blocks ‘\( V_{\text{sd}} \)’ Calculation and ‘\( V_{\text{sq}} \)’ Calculation which are implemented by equations (24).

The block \((dq - \alpha \beta)\) makes the conversion between the synchronous rotating and stationary reference frames and is implemented by equation (3).

So, the calculations blocks replace the classical regulators PI in field control induction motor.
IV. SIMULATION RESULTS

Digital simulation is implemented to display the effectiveness of the backstepping control combined with field oriented control of induction motor. The system parameters of induction motor are given in Appendix. The parameters $k_1, k_2, k_3$ and $k_4$ are chosen as follows: $k_1 = 120$, $k_2 = 100$, $k_3 = 400$ and $k_4 = 30$ to satisfy convergence conditions.

Figure 2, shows the control variable; the stator voltage in $(\alpha, \beta)$ frame, the rotor speed and the rotor flux components in $(d, q)$ frame which present the performance of the backstepping control in the nominal case. It is observed that the rotor speed is very close to the reference one without instabilities effects. It should be noted that the decoupling between the torque and the flux is quite good.
To test the speed evolution of the system, the induction motor is accelerated from standstill to nominal speed (+157 rd/s), afterwards it is decelerated to the inverse rated speed (-157 rd/s) and accelerated again to low speed (30 rd/s). The performances are presented in Figure 3. Note that the decoupling control is very quite maintained with the speed variation. The speed response is merged with the reference one and the flux is very similar to the nominal case. We can find also, that the rotor speed error and rotor flux error given by $z_1$ and $z_2$ converge to zero rapidly.

Thus, in the nominal case, the control gives good quality results. Furthermore, the interest is to verify the robustness of the control with respect to parameter variation. With this aim, we have tested the control according to stator resistance variation.

The results with resistance variation (+50%) between 1.5 and 3.5 s are presented in Figure 4. The speed response is merged with the nominal case (Figure 2). The flux is very similar to the nominal case, we can remark that the increase of the stator resistance amplifies the static error and it appears a small static error in steady condition with respect to nominal case.

From these simulation results, it is obvious that the proposed backstepping controller is quite successful and presents an excellent performance.

Figure 2. Dynamic responses of backstepping control for IM
Figure 3. Backstepping control of IM with rotor speed variation

Figure 4. Backstepping control of IM with stator resistance variation.

V. CONCLUSION

In this paper, we have proposed a backstepping controller for the induction motor with fifth order nonlinear dynamic model which is controlled by primary voltage source. Field-oriented control and...
backstepping design are combined to design the nonlinear model for an induction motor. Step by step control designs are given. The simulation results have demonstrated the effectiveness of our design scheme and have shown that backstepping control can achieve superior performance in comparison to the conventional PI controller.

APPENDIX

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REFERENCES


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