PI Control of Quad-Rotor Unnamed Vehicle Based on Lagrange Approach Modelling

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Abstract-
Aerial robotics is a very exciting research field dealing with a variety of subjects, including the attitude control. This paper deals with the control of a four rotor vertical take-off and landing (VTOL) Unmanned Aerial Vehicle. The paper presents a mathematical model based on the approach of Lagrange for the flight control of an autonomous quad-rotor. It, also, describes the controller architecture which is based on PI regulators. The control method has been simulated in closed loop in different situations. All the calculation stages and the simulation results have been detailed.

Keywords: Quad-rotor, Lagrange approach, Proportional Integral (PI) controller.

1. Introduction

UAVs or ‘Unmanned Aerial Vehicles,’ are defined as aircrafts without the onboard presence of pilots (Gene and al., 1997). UAVs have been used to perform intelligence, surveillance, and reconnaissance missions. The technological promise of UAVs is to serve across the full range of missions. UAVs have several basic advantages over manned systems including increased manoeuvrability, reduced cost, reduced radar signatures, longer endurance, and less risk to crews. Vertical take-off and landing type UAVs exhibit even further manoeuvrability features. Such vehicles are to require little human intervention from takeoff to landing. Unmanned aerial vehicles (UAVs) have potential for full-filling many civil and military applications including surveillance, air pollution monitoring, and area mapping (Castillo and al., 2005).

Unmanned aerial vehicles (UAV) have shown a growing interest thanks to recent technological projections, especially those related to instrumentation.
They made possible the design of powerful systems (mini drones) endowed with real capacities of autonomous navigation at reasonable cost.

In this paper, we studied the behaviour of the quad-rotor. This flying robot presents the main advantage of having quite simple dynamic features (Sahil et al., 2010).

Dynamic modelling is an important step in the development and the control of a dynamic system. In fact, the model allows the engineer to analyze the system, its possibilities and its behaviour depending on various conditions. Moreover, dynamic models are widely used in control design (Noth and al., 2006).

This is especially important for aerial robots where the risk of damage is very high as a fall from a few meters can seriously damage the platform. Thus, the possibility to simulate and tune a controller before implementing it on the real machine is highly appreciable.

This paper is organized as follows: We present a dynamic mathematic model of a quad-rotor model airplane for which the Lagrange-Euler formalism will be used. Then we describe the strategy of tracking control followed by numerical simulation results in various situations, using MATLAB/SIMULINK tools.

2. Quad-rotor dynamic modelling

The quad-rotor (Fig. 1) is a small vehicle with four propellers placed around a main body. The four rotors are used in controlling the vehicle. The rotational speeds of the four rotors are used to control the pitch, roll and yaw attitude of the vehicle.

Fig. 1. Quad-rotor axis system.
The general dynamic model of a quad-rotor has been presented in a number of papers (Gillula and al., 2011; Voos and Bou-Ammar, 2010). It will not be discussed here in details again. All of the movements of quad-rotor can be controlled as shown in Fig. 2.

![Diagram of quad-rotor movements](image)

**Fig. 2.** Flight mode of Quad-rotor.

We consider an inertial frame and a body fixed frame whose origin is in center of mass of the quad-rotor (see Fig. 1). We define the yaw angle, pitch and the roll angle in the following way:

- Roll angle $\phi$ around vector $x$
- Roll angle $\theta$ around vector $y$
- Roll angle $\psi$ around vector $z$

The dynamic model is derived under the following assumptions:

- Structure is rigid and symmetrical;
- The center of mass of vehicle and the body fixed frame origin are assumed to coincide;
- The propellers are rigid in plane.

Under these assumptions. We develop the model of the quad-rotor according to the Lagrange approach:
\[
\begin{align*}
\Gamma_i &= \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} \\
L &= T - V
\end{align*}
\]  

(1)

where:
\( q_i \): Generalized coordinates.
\( \Gamma_i \): Generalized forces.
\( T \): Total kinetic energy.
\( V \): Total potential energy.

The motion equation is given by Lagrangian \( L = T - V \) thus the motion equations of quad-rotor are:

\[
\begin{align*}
\Gamma_\phi &= \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\phi}} \right) - \frac{\partial L}{\partial \phi} = \tau_x \\
\Gamma_\theta &= \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = \tau_y \\
\Gamma_i &= \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\psi}} \right) - \frac{\partial L}{\partial \psi} = \tau_z
\end{align*}
\]  

(2)

The kinetic energy is expressed by \( T = 0.5mv^2 \), where \( m \) is the mass of quad-rotor and \( v \) is the speed of quad-rotor which is given by derivate the vector \( r_x, y, z \). We note that:

\[
r_{x,y,z}(x, y, z) = R(\phi, \theta, \psi) \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}
\]  

(3)

where \( R(\phi, \theta, \psi) \) is the rotational matrix:
Abderrahmen BENBOUALI, Rachid TALEB, Hemza SAIDI, Taieb BESSAAD

\[ R(\phi, \theta, \psi) = \begin{pmatrix}
C_\phi C_\theta & C_\phi S_\theta S_\phi - S_\phi C_\phi & C_\phi S_\theta C_\phi + S_\phi S_\phi \\
S_\phi C_\theta & S_\phi S_\theta S_\phi + C_\phi C_\phi & S_\phi S_\theta C_\phi - S_\phi S_\phi \\
- S_\theta & C_\theta S_\phi & C_\theta C_\phi 
\end{pmatrix} \]

(4)

where \( S_i \) and \( C_i \) represent \( \sin(i) \) and \( \cos(i) \) respectively.

After simplification, we obtain the following expression:

\[
T = \frac{1}{2} I_x (\phi - \psi S_\theta)^2 + \frac{1}{2} I_y (\theta C_\phi + \psi S_\phi C_\theta)^2 + \frac{1}{2} I_z (\theta S_\phi - \psi C_\phi C_\theta)^2
\]

(5)

where \( I_x, I_y \) and \( I_z \) are the moments of inertia according to the x, y and z axis.

Passing now in the potential energy, it’s expressed by:

\[
V = \int x dm (g \cdot S_\phi) + \int y dm (g \cdot S_\phi C_\theta) + \int z dm (g \cdot C_\phi C_\theta)
\]

(6)

Before replacing the equations (5) and (6) in the system (2). We look, first, for the expression of the forces.

In the following system, let us group all the effects of the forces applied to our robot such as thrust, Drag and Gyroscopic effects:

\[
\begin{align*}
\tau_x &= b \cdot l (\Omega_4^2 - \Omega_2^2) + I_r w_x (\Omega_3 + \Omega_1 - \Omega_2 - \Omega_4) \\
\tau_y &= b \cdot l (\Omega_3^2 - \Omega_1^2) + I_r w_x (-\Omega_3 - \Omega_1 + \Omega_2 + \Omega_4) \\
\tau_z &= d (\Omega_1^2 + \Omega_3^2 - \Omega_2^2 - \Omega_4^2)
\end{align*}
\]

(7)

where:

- \( b \) : Thrust coefficient.
- \( d \) : Drag coefficient.
- \( I_r \) : Rotor Inertia.
- \( \Omega_i \) : Angular velocity of motor i.
- \( l \) : Distance between the axis of rotation of rotor and the center of the mass.
- \( w \) : Speed in the fixed frame.
If we replace the system (7) and the equations (5) and (6) in the system (2), we obtain the final model as follow:

\[
\begin{align*}
\ddot{\phi} &= \frac{I_x}{I_x} \left( \Omega_3 + \Omega_1 - \Omega_2 - \Omega_4 \right) + \frac{(I_x - I_z)}{I_x} \ddot{\psi}_\psi + \frac{b \cdot l (\Omega_4^2 - \Omega_2^2)}{I_x} \\
\ddot{\theta} &= \frac{I_y}{I_y} \left( -\Omega_3 - \Omega_1 + \Omega_2 + \Omega_4 \right) + \frac{(I_y - I_x)}{I_y} \ddot{\psi}_\psi + \frac{b \cdot l (\Omega_3^2 - \Omega_1^2)}{I_y} \\
\ddot{\psi}_\psi &= \frac{I_z}{I_z} \ddot{\phi}_\phi + \frac{d (\Omega_4^2 + \Omega_2^2 - \Omega_3^2 - \Omega_1^2)}{I_z}
\end{align*}
\]

(8)

3. PI control design

The PI controllers are widely used in different applications due to their simplicity and easy implementation. To control the quad-rotor in closed loop, we need to represent the dynamics of the rotor. The rotor is a unit constituted by DC motor actuating a propeller via a reducer. The DC motor is governed by the following transfer function:

\[
H(s) = \frac{k}{1 + \tau \cdot s}
\]

(9)

where:

- \( k \): Gain of the motor (rad/s/v).
- \( \tau \): Time constant of the set (motor + reducer).

The simulation design for closed loop control through the MATLAB/Simulink is shown in Fig.3.
Fig. 3. Simulation design with PI controllers.

The robot is piloted through four references (desired value): the dynamics of vertical position reference $Z^*$, the roll reference $\phi^*$, the pitch $\theta^*$ and the yaw angle reference $\psi^*$. These parameters are then sent to the various motors. The first one is delivered to all the motors as the yaw angle reference $\psi^*$. The pitch and the roll are obtained by controlling only two motors (1 and 3 for the pitch, 2 and 4 for the roll). The quad-rotor box (Fig.3) represents the equations clarified in the system (8).

In this paper, the PI controller for the quad-rotor is developed based on the fast response. Using this approach as a recursive algorithm for the control law synthesis, all the calculation stages concerning the tracking errors are simplified.

The PI control is applied to our system as shown in Fig. 3, which will be tuned to determine the optimum response. The PI controller is a closed loop feedback system which will output a control signal and receive feedback. The controller then calculates the difference between the desired attitudes and adjusts the output accordingly by using three parameters (proportional, derived and integral gain (Kim and Shim, 2003).
4. Simulation results

The simulation results are obtained based on the following real parameters shown in Table 1. The quad-rotor system is supplied by a step function for the attitude and vertical axis.

In the first part, we subject the system to three not simultaneous step functions ($10^0$) for the yaw, roll and pitch desired values. We obtain for the (proportional, derivate) controllers (see Table 1) the curves of Fig. 4. We notice that the output angles ($\phi, \theta, \psi$) follow exactly those of references ($\phi^*, \theta^*, \psi^*$).

Now, passing to the second part, where we are going to apply in our system a simultaneous step functions ($\phi^* = 70^0, \theta^* = 10^0, \psi^* = 5^0$). We obtain for various controllers (Table 2) the curves of Fig. 5. As it is planned, the output follows the reference thus we are able to control the quad-rotor by PI regulators.

![Fig. 4. Simulation results with not simultaneous desired values.](image-url)
Fig. 5. Simulation results with simultaneous desired values.

Table 1. Parameters of Quad-rotor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_x$</td>
<td>$6.22 \times 10^{-2}$</td>
<td>Kg.m$^2$</td>
</tr>
<tr>
<td>$I_y$</td>
<td>$7.33 \times 10^{-2}$</td>
<td>Kg.m$^2$</td>
</tr>
<tr>
<td>$I_z$</td>
<td>$9.64 \times 10^{-2}$</td>
<td>Kg.m$^2$</td>
</tr>
<tr>
<td>$I_r$</td>
<td>$4.18 \times 10^{-6}$</td>
<td>Kg.m$^2$</td>
</tr>
<tr>
<td>$l$</td>
<td>0.5</td>
<td>m</td>
</tr>
<tr>
<td>$m$</td>
<td>1</td>
<td>Kg</td>
</tr>
<tr>
<td>$b$</td>
<td>$4.83 \times 10^{-6}$</td>
<td>Kg.m.rad$^{-2}$</td>
</tr>
<tr>
<td>$d$</td>
<td>$2.39 \times 10^{-8}$</td>
<td>Kg.m.s.rad$^{-2}$</td>
</tr>
<tr>
<td>$k$</td>
<td>157.08</td>
<td>rad/s/v</td>
</tr>
<tr>
<td>$\tau$</td>
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<td></td>
</tr>
<tr>
<td>$g$</td>
<td>9.8</td>
<td>m.s$^{-2}$</td>
</tr>
</tbody>
</table>

Table 2. Parameters of PI regulator

<table>
<thead>
<tr>
<th></th>
<th>Not Simultaneous desired values</th>
<th>Simultaneous desired values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll angle $\phi$</td>
<td>$K_p = 0.5$ $K_i = 0.3$</td>
<td>$K_p = 1.5$ $K_i = 0.5$</td>
</tr>
<tr>
<td>Pitch angle $\theta$</td>
<td>$K_p = 0.4$ $K_i = 0.3$</td>
<td>$K_p = 1.5$ $K_i = 0.5$</td>
</tr>
<tr>
<td>Yaw angle $\psi$</td>
<td>$K_p = 12$ $K_i = 10$</td>
<td>$K_p = 12$ $K_i = 10$</td>
</tr>
</tbody>
</table>
5. Conclusion

This paper presented the design of PI controller to control the attitude of quad-rotor system. The mathematical model of the vehicle based on the approach of Lagrange was established in the first section. The second part was reserved to the design of the regulation closed loop including PI regulators and finally, we presented the results of simulation in the last section. These resulting simulink models are ready to be used now by other research.

References


Qing Wang, Jun-Wei Wang, Yao Yu, Chang-Yin Sun, 2014. Robust attitude control of an indoor micro quadrotor with input delay. IEEE


